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INVENTORY MODEL (M. T.) EXPONENTIAL BACKORDER COST RANDOM SUPPLY AND CONTINUOUS LEAD TIME SERIES 3

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Abstract
We derived the inventory cost for Inventory Model (M.T) in which the supply is random, backorder costs is exponential and lead time is continuous by making use of the inventory costs derived in series 2 for the same random supply but for constant lead time.
Each factor of the inventory cost is then averaged over the states of lead times. Lead time is assumed to be a gamma variate.

Introduction
This paper makes use of the inventory costs from model (M.T) where the lead time is continuous, supply is random and backorder costs is exponential in series 2 and average this cost over the states of the lead time. The lead time is a gamma variate.

Literature Review
Nasir Paknejad and Affison, (2012) utilized an EOQ model with non-linear holding costs.
Singh, (2013) deals with an inventory model with non-instantaneous receipt under trade credit and time dependent rate and exponential demand rate.

Zipkin (2006) treats both fixed and random lead times and examines both stationery and limiting distributions under different assumptions.

Hadley and Whitin (1972) extensively developed the (M.T) model for constant lead time and linear costs.

The probability distribution function of L is given as:

$$H(L) = \frac{\exp(\propto L) L^{k-1} \propto^k}{\lceil k} \qquad k, L, \ \propto > 0$$

Demand in the lead time L follows a normal distribution g $(\mu, \sigma^2 T)$

$$g\ (\mu, \sigma^2 T) = \frac{1}{\sqrt{2\pi\,\sigma^2 t}}\, exp - \left(\frac{x-\mu}{\sqrt{\sigma^2 t}}\right) - \infty < x > \infty$$

The inventory cost when the supply was random and lead times were constant is given from equation 10 series 2 as

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$$C = \frac{(RC+S)}{T} + \frac{hcv}{\mu} - hc\frac{(DL+DT)}{2} + hc(G_{11}(T+L) - G_{11}(L)) + \frac{1}{T}(G_{14}(T+L) - G_{14}(L))$$

Where from series 2 equation 7

$$G_{11}(T+L) = \frac{\mu^v esp\left(-(T+L)\left(D\mu - \frac{\mu^2\sigma^2}{2}\right)\right)}{\Gamma v}\left(\frac{\sigma^2}{4D^3} + \frac{D(T+L)^2}{2} + \frac{\sigma^2}{2D^2} - (T+L)\right)$$

$$\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-2}{2}} \frac{(v-1)!}{\mu^z(v-z)!}\binom{v-z-i}{i}(D(T+L) - \mu\sigma^2(T+L))^{v-z-2i}\left(\frac{\sigma^2(T+L}{2}\right)^i + \frac{1}{2D}$$

$$\sum_{z=1}^{v+2}\sum_{i=0}^{\frac{v+2-z}{2}} \frac{(v+1)!}{\mu^z(v+2-z)!}\binom{v+2-z-i}{i}(D(T+L) - \mu\sigma^2(T+L)^{v+2-z-2i}\left(\frac{\sigma^2(T+L}{2}\right)^i\Bigg]$$

$$+\frac{\frac{\sigma 2}{2}(T+L)\mu^v esp(-(T+L))\left(D\mu - \frac{\mu^2\sigma^2}{2}\right)}{\Gamma v}\left[\left(T - \frac{\sigma^2}{D^2}\right)\right]\sum_{i=0}^{\frac{v-1}{2}}\binom{v-1-i}{i}$$

$$(D(T+L) - \mu\sigma^2(T+L))^{v-1-2i}\left(\frac{\sigma^2(T+L)}{2}\right)^i - \frac{1}{D}\sum_{i=0}^{\frac{v}{2}}\binom{v-1}{i}(D(T+L) - \mu\sigma^2(T+L)^{v-2i}$$

$$\left(\frac{\sigma^2(T+L)}{2}\right)^i\Bigg] - \frac{\mu^v esp(-T+L)\left(D\mu - \frac{\mu^2\sigma^2}{2}\right)}{4D^3\,\Gamma v}\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-2}{2}}\frac{(v-1)^i}{\left(\mu - \frac{2D}{\sigma^2}\right)^z(v-z)!}\binom{v-z-1}{i}$$

$$(D(T+L) - \mu\sigma^2(T+L))^{v-z-2i}\left(\frac{\sigma^2(T+L)}{2}\right)^i$$

Substituting T+L for T in equation (9) series of 2, we have

$$G_{14}(T+L) = 2b_1\mu_v esp\left[\left(\frac{\sigma^2 b_2 T + Tb_2}{2D_2}\right) + \left(\mu + \frac{b_2}{D}\right)\left(\frac{\sigma^2 T}{2}\left(\mu + \frac{b_2}{D}\right) - DT\right)\right]$$

$$\frac{esp\left[-L\left(\mu + \frac{b^2}{D}\right)\left(D - \frac{\sigma^2}{2}\left(\mu + \frac{b^2}{D}\right)\right) - \left(\frac{\sigma^2 b_2^2 + b_2}{2D^2}\right)\right]}{b_2\,\Gamma v(\sigma^2 b_2^2 + 2D^2 b_2)}\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-z}{2}}\frac{(v-1)^i}{\left(\mu - \frac{b_2}{D}\right)^z(v-z)!}$$

$$\binom{v-z-1}{i}(D(T+L) - \mu\sigma^2(T+L))^{v-z-2i}\left(\frac{\sigma^2(T+L)}{2}\right)^i + \frac{b_1}{b_2}\frac{\mu^v}{\Gamma(v)}$$

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$$\mathrm{esp}\left(\frac{\mu^2\sigma^2 T}{2} - D\mu T\right)\mathrm{esp}\left(-L\left(D\mu - \frac{\mu^2\sigma^2}{2}\right)\right)\sum_{i=0}^{\frac{v}{2}}\binom{v-1}{i}\left(D(T+L) - \mu\sigma^2(T+L)\right)^{v-2i}$$

$$\left(\frac{\sigma^2(T+L)}{2}\right)^i - D(T+L)\sum_{i=0}^{\frac{v-1}{2}}\binom{v-1-i}{i}\left(D(T+L) - \mu\sigma^2(T+L)\right)^{v-1-2i}\left(\frac{\sigma^2(T+L)}{2}\right)^i\Bigg]$$

$$-\frac{\sigma^2 b_1 \mu^v}{D(\sigma^2 b_2 + 2D^2)}\mathrm{esp}\frac{\left(\frac{\mu^2\sigma^2 T}{2} - D\mu T\right)}{\Gamma(v)}esp\left(-L\left(D\mu - \frac{\mu^2\sigma^2}{2}\right)\right)\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-z}{2}}\frac{(v-1)!}{\left(\mu - \frac{2D}{\sigma^2}\right)^z (v-z)!}$$

$$\binom{v-z-1}{i}\left(D(T+L) - \mu\sigma^2(T+L)^{v-z-2i}\left(\frac{\sigma^2}{2}(T+L)\right)^i\right) \qquad (4)$$

Simplifying G_{11} (T+L) equation (2)

$$G_{11}(T+L) = \mu^v esp\frac{\left(-(T+L)\left(D\mu - \frac{\mu^2\sigma^2}{2}\right)\right)}{\Gamma(v)}\left[\left(\frac{\sigma^2}{4D^3} + \frac{DT^2}{2} + \frac{\sigma^2}{2D^2} - T\right) + L\left((DT-1) + \frac{DL^2}{2}\right)\right.$$

$$\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-z}{2}}\sum_{w=0}^{v-z-1}\frac{(v-1)!}{\mu^z(v-2)!}\binom{v-z-i}{i}\binom{v-z-i}{w}(D-\mu\sigma^2)^{v-z-2i}\left(\frac{\sigma^2}{2}\right)^1 T^{v-z-i-w}L^w + \frac{1}{2D}$$

$$\sum_{z=1}^{v+1}\sum_{i=0}^{\frac{v-1-z}{2}}\sum_{w=0}^{v+1-z-i}\frac{v!}{\mu^z(v+2-z)!}\binom{v+1-z-i}{i}\binom{v+1-z-i}{w}(D-\mu\sigma^2)^{v-z-2i}$$

$$\left.\left(\frac{\sigma^2}{2}\right)^i T^{v-z-i-w}L^w\right] + \frac{\sigma^2\mu^v esp(-(T+L)\left(D\mu - \frac{\mu^2\sigma^2}{2}\right)}{\Gamma(v)}(T+L)\left(T - \frac{\sigma^2}{D^2}\right)$$

$$\sum_{i=0}^{\frac{v-1}{2}}\sum_{w=1}^{v-1-i}\binom{v-1-i}{i}\binom{v-1-i}{w}(D-\mu\sigma^2)^{v-1-2i}\left(\frac{\sigma^2}{2}\right)^i T^{v-i-w}L^w - \frac{(T+L)}{D}$$

$$\left.\sum_{i=0}^{\frac{v}{2}}\sum_{w=0}^{v-i}\binom{v-i}{i}\binom{v-i}{w}(D-\mu\sigma^2)^{v-i-w}\left(\frac{\sigma^2}{2}\right)^i T^{v-i-w}L^w\right] - \frac{\mu^v esp(-(T+L))\left(D\mu - \frac{\mu^2\sigma^2}{2}\right)}{4D^3\sqrt{(v)}}$$

$$\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-z}{2}}\sum_{w=0}^{v-z-2}\frac{(v-1)!}{\left(\mu - \frac{2D}{\sigma^2}\right)^z(v-z)!}\binom{v-z-i}{i}\binom{v-z-i}{w}(D-\mu\sigma^2)^{v-z-2i}$$

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$$\left(\frac{\sigma^2}{2}\right)^1 T^{v-z-2-w} L^w$$

$$Multiplying\ by\ H\ (L) = \frac{esp(-\propto L) L^{k-1} \propto^k}{\lceil(k)}$$

$$and\ noting\ that\ \int_0^\infty esp\ (-\propto L) L^y\, dY = \sqrt{(Y+1)} / \propto^{y+1}$$

$$Then\ G_{15}(T) = \frac{\mu^v \propto^k esp\left(-T\left(D\mu - \frac{\mu^2\sigma^2}{2}\right)\right)}{\lceil(v)\ \lceil(k)}$$

$$\left[\sum_{z=1}^{v} \sum_{i=0}^{\frac{v-z}{2}} \sum_{w=0}^{v-z-i} \frac{(v-1)!}{\mu(v-z)!} \binom{v-z-i}{i} \binom{v-z-i}{w} (D-\mu\sigma^2)^{v-z-2i} \left(\frac{\sigma^2}{2}\right) T^{v-z-i-w} L^w\right.$$

$$\left(\left(\frac{\sigma^4}{4D^3} + \frac{DT^2}{2} + \frac{\sigma^2}{2D^2} - T\right) \frac{\sqrt{(w+k)}}{\left(D\mu - \frac{\mu^2\sigma^2}{2} + \propto\right)^{w+k}} + (DT-1) \frac{\sqrt{(w+k+1)}}{\left(D\mu - \frac{\mu^2\sigma^2}{2} + \propto\right)^{w+k+1}} + \frac{D}{2}\right.$$

$$\left.\frac{\sqrt{(w+k+2)}}{\left(D\mu - \frac{\mu^2\sigma^2}{2} + \propto\right)^{w+k+2}}\right) + \frac{1}{2D} \sum_{z=1}^{v+1} \sum_{i=0}^{\frac{v+1-z}{2}} \sum_{w=0}^{v+1-z-2} \frac{v!}{\mu^z (v+1-z)!} \binom{v+1-z-i}{i}$$

$$\left.\binom{v+1-z-i}{\mathrm{w}} (\mathrm{D} - \mu^2\sigma^2)^{v-z-2i} \left(\frac{\sigma^2}{2}\right)^i \mathrm{T}^{v+1-z-i-w} \frac{\sqrt{(w+k)}}{\left(D\mu - \frac{\mu^2\sigma^2}{2} + \propto\right)^{w+k}}\right]$$

$$+\frac{\sigma^2}{2} \mu^v esp\left(-T\left(D\mu - \frac{\mu^2\sigma^2}{2}\right)\right) \propto^k \sum_{i=0}^{\frac{v-1}{2}} \sum_{w=0}^{v-1-i} \binom{v-1-i}{i} \binom{v-1-i}{w} (D-\mu\sigma^2)^{v-1-2i}$$

$$\left(\frac{\sigma^2}{2}\right)^i T^{v-1-i-w} \left(\left(T^2 - \frac{\sigma^2 T}{D^2}\right)\right) \frac{\sqrt{(w+k)}}{\left(D\mu - \frac{\mu^2\sigma^2}{2} + \propto\right)^{w+k}} + \left(T - \frac{\sigma^2}{D^2}\right) \frac{\sqrt{(w+k+1)}}{\left(D\mu - \frac{\mu^2\sigma^2}{2} + \propto\right)^{w+k+1}}$$

$$-\frac{1}{D} \sum_{i=0}^{\frac{v}{2}} \sum_{w=0}^{v-i} \binom{v-i}{i} \binom{v-i}{w} (D-\mu\sigma^2)^{v-i-w} \left(\frac{\sigma^2}{2}\right)^i T^{v-1-i-w} \left(\frac{T\sqrt{(w+k)}}{\left(D\mu - \frac{\mu^2\sigma^2}{2} + \propto\right)^{w+k}}\right.$$

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$$+\frac{\sqrt{(w+k+1)}}{\left(D\mu-\frac{\mu^2\sigma^2}{2}+\infty\right)^{w+k+1}}\Bigg)\Bigg]-\frac{\mu^v \propto^k esp\left(-T\left(D\mu-\frac{\mu^2\sigma^2}{2}\right)\right)}{4D^3\Gamma(v)\Gamma(k)}\sum_{z=1}^{v+1}\sum_{i=0}^{\frac{v-2}{2}}$$

$$\sum_{w=0}^{v-z-i}\frac{(v-1)!}{\left(\mu-\frac{2D}{\sigma^2}\right)^z(v-z)!}\binom{v-z-i}{i}\binom{v-z-i}{w}(D-\mu\sigma^2)^{v-z-2i}\,T^{v-z-i-w}$$

$$\frac{\sqrt{(w+k)}}{\left(D\mu-\frac{\mu^2\sigma^2}{2}+\propto\right)^{w+k}} \tag{6}$$

Simplifying G_{14} (T+L) equation (4)

$$G_{14}(T+L)=2Db_1\mu^v esp\left(T\left(\left(\frac{\sigma^2 b_2}{2D^2}+b_2\right)+\left(\mu+\frac{b_2}{D}\right)\sigma^2\left(\mu+\frac{b_2}{D}\right)-D\right)\right)$$

$$\frac{esp\left[-L\left(\mu+\frac{b^2}{D}\right)\left(D-\sigma^2\left(\mu+\frac{b^2}{D}\right)\right)-\left(\frac{\sigma^2 b_2}{2D^2}+b_2\right)\right]}{b_2\left(\sigma^2 b_2^2+2D^2 b_2\right)\Gamma(v)}\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-z}{2}}\sum_{w=0}^{v-z-i}\frac{(v-1)^i}{\left(\mu-\frac{b_2}{D}\right)^2(v-z)!}$$

$$\binom{v-z-i}{i}\binom{v-z-i}{w}(D-\mu\sigma^2)^{v-z-2i}\left(\frac{\sigma^2}{2}\right)^i T^{v-z-i-w}L^w+\frac{b_1}{b_2}\frac{\mu^v}{\Gamma(v)}$$

$$esp\left(\frac{\mu^2\sigma^2 T}{2}-D\mu T\right)esp\left(-L\left(D\mu-\frac{\mu^2\sigma^2}{2}\right)\right)\left[\sum_{z=0}^{\frac{v}{2}}\sum_{w=0}^{v-2}\binom{v-2}{2}\binom{v-2}{w}^{v-2}\left(\frac{\sigma^2}{2}\right)^i T^{v-2-w}L^w\right.$$

$$\left.-D(T+L)\sum_{z=0}^{\frac{v-1}{2}}\sum_{w=0}^{v-1-z}\binom{v+1-2}{2}\binom{v+1-2}{w}(D-\mu\sigma^2)^{v-1-z}\left(\frac{\sigma^2}{2}\right)^i T^{v-2-w}L^w\right]$$

$$-\frac{\sigma^2 b_1\mu^v}{D(\sigma^2 b_2+2D^2)}\,esp\frac{\left(\frac{\mu^2\sigma^2 T}{2}-D\mu T\right)}{\Gamma(v)}\,esp\left(-L\left(D\mu-\frac{\mu^2\sigma^2}{2}\right)\right.\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-1}{2}}\sum_{w=0}^{v-z-i}\frac{(v-1)!}{\left(\mu-\frac{2D}{\sigma^2}\right)^z(v-z)!}$$

$$\binom{v-z-i}{i}\binom{v-z-i}{w}(D-\mu\sigma^2)^{v-z-2i}\left(\frac{\sigma^2}{2}\right)^i T^{v-z-i-w}L^w$$

Multiplying by H (L) and noting that

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$$\int_{\cup}^{\infty} esp\,(-\infty L)L^{y}\,dY = \sqrt{(Y+1)}/\infty^{y+1} \qquad and$$

$$the\ integrating\ \int_{0}^{\infty} G_{14}\,(T+L)H(L)dL$$

We have

$$G_{17}(T) = \frac{2Db_1\mu^{v} esp\left(T\left(\frac{\sigma^2 b_2}{D}+b_2\right)+\left(\mu+\frac{b_2}{D}\right)\left(\sigma^2\left(\mu+\frac{b_2}{D}\right)-D\right)\right)}{b_2\left(\sigma^2\, b_2^{\,2}+2D^2 b_2\right)\lceil(v)\,\lceil(k)}$$

$$\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-z}{2}}\sum_{w=0}^{v-z-i}\frac{(v-1)!}{\left(\mu-\frac{b_2}{D}\right)^{2}(v-z)!}\binom{v-z-i}{i}\binom{v-z-i}{w}(D-\mu\sigma^2)^{v-z-2i}\left(\frac{\sigma^2}{2}\right)T^{v-z-i-w}$$

$$\frac{\sqrt{(w+k)}}{\left[\left(\mu+\frac{b_2}{D}\right)\left(D-\frac{\mu\sigma^2}{2}-\frac{b_2\sigma^2}{2D}\right)-\frac{\sigma^2 b_2^{\,2}}{2D^2}-b_2\propto\right]^{w+k}}-\frac{b_1}{b_2}\frac{\mu^{v}}{\lceil(v)\,\lceil(w)}esp\,\left(\frac{\mu^2\sigma^2 T}{2}-D\mu T\right)$$

$$\sum_{i=0}^{\frac{v}{2}}\sum_{w=0}^{v-z}\binom{v-i}{i}\binom{v-i}{w}\,(D-\mu\sigma^2)^{v-2i}\left(\frac{\sigma^2}{2}\right)^{i}T^{v-i-w}\frac{\sqrt{(w+k)}}{\left(D\mu-\frac{\mu^2\sigma^2}{2}+\propto\right)^{w+k}}-D$$

$$\sum_{i=0}^{\frac{v-1}{2}}\sum_{w=0}^{v-1-i}\binom{v-1-i}{i}\binom{v-1-i}{w}\,(D-\mu\sigma^2)^{v-1-i}\left(\frac{\sigma^2}{2}\right)^{i}T^{v-i-w}$$

$$\left(\frac{T\sqrt{(w+k)}}{\left(D\mu-\frac{\mu^{2\sigma^2}}{2}+\propto\right)^{w+k}}+\frac{\sqrt{(w+k-1)}}{\left(D\mu-\frac{\mu^{2\sigma^2}}{2}+\propto\right)^{w+k}}\right)-\frac{\rho^2 b_1\mu^{v}\infty^{k}}{D(\sigma^2 b_2+2D^2)}\frac{esp\left(\frac{\mu^2\sigma^2 T}{2}-D\mu T\right)}{\lceil(v)\,\lceil(k)}$$

$$\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-z}{2}}\sum_{w=0}^{v-z-i}\frac{(v-1)!}{\left(\mu-\frac{2D}{\sigma^2}\right)^{z}(v-z)!}\binom{v-z-i}{i}\binom{v-z-i}{w}(D-\mu\sigma^2)^{v-z-2i}\left(\frac{\sigma^2}{2}\right)T^{v-z-i-w}$$

$$\frac{T\sqrt{(w+k)}}{\left(D\mu-\frac{\mu^{2\sigma^2}}{2}+\propto\right)^{w+k}}$$

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Hence applying the above integrals the inventory cost when the supply is random, and the lead times are continuous and the cost of a backorder is an exponential function of the length of time is

$$C = \frac{Rc + S}{T} + \frac{hc: v}{\mu} + hc\left(\frac{Dk}{\propto} + \frac{DT}{k}\right) + \left(G_5(T) - G_{15}(0)\right)$$

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